Centre Interfacultaire Bernoulli - CIB

Introduction to Partition Functions

Book of abstracts

16 to 27 July 2018 - EPFL, room BI A0 448

Alexander Barvinok

2 lectures: The interpolation method.

I will describe a general approach which allows one to approximate the partition function in a complex domain provided it does not have zeros in a slightly larger domain. Typically, this way we obtain quasi-polynomial algorithms since the logarithm of the partition function gets approximated by a polynomial of logarithmic degree, although in many important cases the method can be sharpened to produce genuinely polynomial time algorithms, as shown by Patel and Regts. Examples include permanents of matrices and tensors, the independence polynomial of a graph, graph homomorphism partition functions and their versions. The method can be considered as an algorithmic version of the Lee-Yang approach to the phase transition in statistical physics: we approximate the logarithm of the partition function at a finite temperature by a low degree Taylor polynomial computed at an infinitely high temperature, and the approximation is guaranteed to work fine as long as there is no phase transition on the way.

Michael Chertkov & Pascal Vontobel

Lecture 1

- Introduction to factor graphs and the sum-product algorithm
- Bethe approximation
- Graph-cover characterization of the Bethe partition function

Lecture 2

- Gauge transformation
- Factor graph transformation preserving partition function (Part 1)
  - Fixed point of the Bethe free energy (log partition function) as a special choice of gauges
  - Loop calculus and series

Lecture 3
- Bethe approximation of the permanent of a non-negative matrix
- Factor graph transforms that preserve the partition function (Part 2)

Lecture 4
- Beyond belief propagation
- Three methods for approximate estimation of the partition function:
  - Sampling of loop series
  - Gauge transformations guaranteeing a lower bound on the partition function
  - Bucket renormalization
- Conclusion and discussion of future challenges

Peter Csikvari

2 lectures: Applications of graph covers.

Graph covers provides an efficient way to connect a partition function of a graph with the corresponding Bethe partition function. Building on this observation I will sketch how to give a lower bound on the number of perfect matchings of a graph (permanents of a matrix) using graph covers, and also give some elementary observations about the number of independent sets and the partition function of the Ising model. In the second lecture, I will give an account into the work of Nicholas Ruozzi that connects the use of correlation inequalities with graph covers.

Florent Krzakala & Lenka Zdeborova

4 lectures: Recent rigorous results on proving the replica free energy and gentle intro to replica symmetry breaking.

In this lecture we will highlight the spirit of the replica method for computing free energies. Then we will focus of recent progress on rigorous proofs of the free energy in model for Bayes optimal inference. In a second part we will introduce the concept of replica symmetry breaking (RSB) and overview the classes of models with examples for which RSB is conjectured to give exact limits for the corresponding free energies.

Alex Samorodnitsky

2 lectures: Scaling.

Some partition functions can be approximated by applying an efficiently computable transformation (i.e., scaling) to the underlying object, so that the following conditions are satisfied: (1) We know how the value of the partition function changes. (2) We have a reasonably good idea about the behavior of the partition function on the scaled objects. We will demonstrate this on some examples, focusing on probably the simplest one, that is approximating the permanent of a matrix with nonnegative entries.

Daniel Stefankovic & Eric Vigoda

Lecture 1: Counting and sampling connections.

Detailing connections between counting and sampling problems, focusing on improved reduction from approx. counting to sampling.

Lecture 2: MCMC convergence.

Techniques for analyzing the convergence rate of Markov chains, emphasizing coupling and canonical paths techniques.

Lecture 3: Permanent.

Detailing JSV's FPRAS for the permanent.

Lecture 4: Gadgets and hardness.

Explaining Sly's gadget and its key properties, and how its used for hardness of approximate counting results.

Nikhil Srivastava

2 lectures: Stable Polynomials and Lower Bounds on the Permanent.

We will survey the theory of real stable polynomials in many variable, with an emphasis on characterizing linear operators which preserve stability, mainly following the work of Borcea and Branden. We will then explain Gurvits' elegant proof of van der Waerden's conjecture using some elements of this theory.

Piyush Srivastava & Yitong Yin

Correlation decay and approximate counting.

In the recent past, the correlation decay method has helped establish some of the tightest known connections between phase transitions and computational complexity theory: it underlies the algorithmic component of the fact that the so-called uniqueness phase transition exactly determines the computational complexity of approximating the partition function (or sampling from the Gibbs distribution) of models such as independent sets (the hard core lattice gas) and the anti-ferromagnetic Ising model.

Although the importance of decay of correlation in the analysis of Gibbs sampling and belief propagation was well established in the literature, the first direct applications of the method to counting appear to have been made by Bandhyopadhyay and Gamarnik (2005) and Weitz (2006). Weitz in particular gave a deterministic approximate counting algorithm that worked up to the uniqueness phase transition threshold of the hard core model, something that had not then been achieved even using randomised Markov chain Monte Carlo methods that are seen as the workhorse for the approximation of partition functions.

This series of talks will provide an overview of the correlation decay method, and survey some recent work that extending the method. The schedule is as follows:

Lecture 1: Introduction to the correlation decay method.
Lecture 2: Correlation decay for distributed counting.
Lecture 3: Beyond bounded degree graphs.
Lecture 4: Correlation decay, zeros of polynomials, and the Lovász local lemma.

<u>**Mohit Singh**</u>

Algorithms for maximum entropy convex programs and relationship to counting problems.

We will outline the role of convex optimization in obtaining algorithms for maximum entropy programs and its relationship to counting problems. We will discuss convex duality as well as polarity that play an important role in establishing this relationship. We will also discuss the classical ellipsoid algorithm for convex optimization and pitfalls that arise while applying it to the maximum entropy programs as well as some techniques to deal with them.