**Abstract:**

The matrix equations $M^2 = 0$ are quadratic, so to derive the linear equation $\text{Trace}(M) = 0$ from them requires nonalgebraic operations. Are there corresponding "surprising" equations implied by the matrix equation $XY = YX$? This question was posed in the '60s, and still nobody knows. Even the (normalized) volume of this space \{(X,Y) : XY = YX\} is very difficult to compute for large matrices, and until recently was only known to start 1, 3, 31, 1145.

I'll talk about a bunch of related spaces of matrices, some of which are provably harder and some easier to understand than the commuting scheme \{(X,Y) : XY = YX\}, and the volumes of these spaces. Then I'll explain how physicists came up with the same set of numbers from a statistical mechanical model (making them much easier to compute), and why they are indeed the same.

Some of this work is joint with Paul Zinn-Justin.