Limits of finite graphs: Functions, groups and measurable equivalence relations.

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November 10, 2006

More than thirty years ago Kaplansky asked the following question: Is it true that for any group $\Gamma$ and finite field $F$ the group algebra $F[G]$ is directly finite, that is $AB = 1$, implies $BA = 1$? If the group is finite a positive answer follows from the fact that for finite dimensional matrices injectivity and surjectivity are equivalent. It turns out that if a group is well-approximable with finite graphs then the answer is still yes. These so-called sofic groups were introduced by Gromov and Weiss. In the first part of the course we consider these groups and some famous conjectures (the Determinant Conjecture, the Embedding Conjecture, the Gottschalk Conjecture) which can be proved for sofic groups.

In the second part of the mini-course we consider limits of finite graphs in general. Following Lovasz and Szegedy we prove the famous Szemeredi Regularity Lemma using limit objects (measurable functions) of dense graphs. Then we consider the bounded vertex degree case and show that the right limit objects are measurable equivalence relations. We give graph theoretical interpretation for the cost and the first $L^2$-Betti numbers, introduced by Levitt and Gaboriau and the rank gradient, introduced by Lackenby for residually finite groups.