Recent Developments on the Arithmetic of Special Values of L-functions
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Fabrizio Andreatta
Università Statale di Milano

Triple product p-adic L-functions associated to finite slope p-adic families of modular forms.

I will explain how one can extend the methods developed together with Iovita and Pilloni to construct p-adic deformations of finite slope de Rham classes. As an application I will define p-adic L-functions attached to a triple of p-adic finite slope families of modular forms. This is joint work with Adrian Iovita.

Michele Fornea
McGill University

Twisted triple product p-adic L-functions and Hirzebruch-Zagier cycles

For $L/F$ a quadratic extension of totally real number fields and $p$ an unramified prime, we construct a p-adic L-function interpolating the central values of the twisted triple product L-functions attached to nearly ordinary families of Hilbert modular forms. When $L$ is a real quadratic field and $p$ is a split prime, we prove a p-adic Gross-Zagier formula relating the values of the p-adic L-functions outside the range of interpolation to the syntomic Abel-Jacobi image of generalized Hirzebruch-Zagier cycles. The formula, in combination with a recent work of Y. Liu, gives a result in the spirit of the Bloch-Kato conjecture for some p-adic representation attached to elliptic curves $A/L$ and $E/Q$. This is joint work with Iván Blanco-Chacón.

Lennart Gehrmann
University Duisburg-Essen

Derived Hecke algebra and automorphic L-invariants

Hilbert modular cusp forms of parallel weight two only show up in the middle degree cohomology of the appropriate locally symmetric space, whereas the analogous automorphic forms over number fields of arbitrary sign (e.g. Bianchi modular forms) show up in several degrees. In this talk I define automorphic L-invariants for each of these degrees and show that Venkatesh’s conjecture on derived Hecke algebras implies a precise relationship between these L-invariants.

Yifeng Liu
Northwestern University

Bloch-Kato conjecture for Rankin-Selberg motives, I.

In this talk, I will formulate explicitly the Bloch-Kato conjecture for sym-
plectic conjugate self-dual motives of $GL(m)^*GL(n)$ over CM fields, which relates Selmer groups and Rankin-Selberg $L$-functions in the central critical case. When $m=n+1$, a theorem concerning rank 0 and rank 1 will be stated, which is the main theme of an ongoing joint work with Y. Tian, L. Xiao, W. Zhang, and X. Zhu. Then I will introduce various Shimura varieties of unitary type which we use. If time permits, I will state an arithmetic level raising result for odd dimensional unitary Shimura varieties, which is a key ingredient in our approach.

David Loeffler
University of Warwick

Heegner points in Coleman families

Using Heegner points, or more general Heegner cycles, one can build Galois cohomology classes for a modular form twisted by an anticyclotomic Grössencharacter of an imaginary quadratic field. I will report on a joint project with Jetchev and Zerbes in which we show that these classes naturally interpolate into a 2-parameter family, with the modular form varying through a Coleman family, and the ring class character also varying. (This extends earlier results due to many authors including Howard, Disegni, and Kobayashi.)

I will also explain a general programme to obtain new Euler systems by specialising such families at critical-slope Eisenstein series; in this Heegner-point setting this should give a new construction of the elliptic units, and potentially a generalisation of elliptic units to higher-degree CM fields.

Victor Rotger
Universitat Politècnica de Catalunya

Stark-Heegner points and the Euler system of diagonal cycles

Stark-Heegner points are conjectural substitutes for Heegner points when the imaginary quadratic field of the theory of complex multiplication is replaced by a real quadratic field $K$. They are constructed analytically as local points on elliptic curves with multiplicative reduction at a prime $p$ that remains inert in $K$, but are conjectured to be rational over ring class fields of $K$ and to satisfy a Shimura reciprocity law describing the action of $G_K$ on them. This amounts to claiming that any linear combination of Stark-Heegner points weighted by the values of a ring class character of $K$ should belong to the corresponding piece of the Mordell-Weil group over the associated ring class field. In this lecture I will report on joint work with H. Darmon, where we show that such linear combinations arise from the localisation at primes
above $p$ of global classes in the idoneous pro-$p$ Selmer group. The proof rests on a direct comparison between Stark-Heegner points and the generalised Kato classes arising from the Euler system of diagonal cycles.

Shrenik Shah  
Columbia University

*Geometry and representation theory of Archimedean regulator pairings*

We describe some connections between Archimedean regulator pairings and ideas in geometry and representation theory, with an emphasis on Picard modular surfaces and the Shimura fourfolds associated to GU$(2, 2)$. For instance, the relative Fourier-Mukai transform will arise as well as distinction and multiplicity one for representations of $p$-adic groups. This is joint work with Aaron Pollack.

Florian Sprung  
Princeton University

*The Birch and Swinnerton-Dyer formula in the non-ordinary case*

We present a proof of the main conjectures in the non-ordinary case, for elliptic curves, and for weight two modular forms.

Ye Tian  
Academy of Mathematics and System Science

*A converse theorem of Gross-Zagier and Kolyvagin: CM case*

Let $E$ be a CM elliptic curve over rationals and $p$ an odd ordinary prime. Assume that $p$-Selmer group of $E$ has corank one, we show that the analytic rank of $E$ is also one. This is joint work with Ashay Burungale.

Yichao Tian  
Universität Bonn

*Bloch-Kato conjecture for Rankin-Selberg motives II*

This is a continuation of Yifeng Liu’s talk on the progress of our on-going project with Liang Xiao, Wei Zhang and Xinwen Zhu. In this talk, I will focus on the case of $U(2)^*U(3)$ motives. I will start with explaining the proof of the level raising result in the case of $U(2)$ Shimura varieties, perhaps already stated in Yifeng’s talk. Then I will explain how to deduce one side implication of the Bloch-Kato conjecture for $U(2)^*U(3)$ motives in the rank 0 case under some additional assumptions. A key ingredient is the generic Tate conjecture for $U(3)$ Shimura varieties over finite fields proved recently.
by Liang Xiao and Xinwen Zhu.