Concluding conference of the program
Real and Tropical Algebraic Geometry

Abstracts

F. Bihan, *Fewnomial bounds on Betti numbers of hypersurfaces.*

We use stratified Morse theory and recent fewnomial bounds on the number of positive solutions of polynomial systems to derive fewnomial bounds on the total Betti number of hypersurfaces of the positive orthant. This is a joint work with Frank Sottile.

E. Brugallé, *Recursive formulas for Welschinger invariants.*

I will explain how to obtain recursive formulas for Weslchinger invariants in dimension 2 and 3 using the "floor decomposition" technic. This talk includes joint works with Lucia Lopez de Medrano and Grigory Mikhalkin.

F. Catanese, *Real singular Del Pezzo surfaces and threefolds fibred by rational curves.*

Let $W \to X$ be a real smooth projective threefold fibred by rational curves. Kollár proved that if $W(\mathbb{R})$ is orientable a connected component N of $W(\mathbb{R})$ is essentially either a Seifert fibred manifold or a connected sum of lens spaces. Our Main Theorem gives sharp estimates on the number and the multiplicities of the Seifert fibres and on the number and the torsions of the lens spaces when X is a geometrically rational surface. Our results answer in the affirmative three questions of Kollár, and can be interpreted in terms of the geometric structure which N admits. For instance, when N is Seifert fibred over a base orbifold F, our results imply the following generalization of Comessatti's theorem on real rational surfaces: F cannot be simultaneously orientable and of hyperbolic type. We derive our Theorem from a careful study of global configurations of singularities on real singular Del Pezzo surfaces with only Du Val singularities.

A. Degtyarev, *Real elliptic surfaces and real trigonal curves of type I* (joint with I. Itenberg).

We attempt to study/classify real Jacobian elliptic surfaces of type I or, equivalently, separating real trigonal curves in geometrically ruled surfaces. We reduce the problem to a simple graph theoretical question and, as a result, obtain a characterization and complete classification (quasi-simplicity) in the case of rational base. (The results are partially interlaced with those by V. Zvonilov.) As a by-product, we obtain a criterion for a trigonal curve of type I to be isotopic to a maximally inflected one.

S. Finashin, *Chiral deformation classification of real cubic fourfolds* (joint work with V. Kharlamov).

Cubic fourfolds are remarkably related to K3 surfaces and share some of their fundamental properties: Torelli theorem, a version of epimorphism of periods, etc. We obtained deformation classification of real cubic fourfolds following the same strategy as for K3 surfaces. Namely, we proved that the coarse deformation classes are determined by the (+1)- and (-1)-eigensublattices in the middle homology. These sublattices turn out to be certain twists of the eigensublattices of K3 surfaces. Furthermore, we upgraded the coarse deformation classification of cubic fourfolds to the chiral deformation classification. Namely, the classification problem was reduced to arithmetics of the eigenlattices: a question about certain symmetries of their Coxeter graphs. In all but a few cases this question was resolved.

V. Fock, *Triangles.*

Triangle of size N is a collection of numbers associated to points of a plane $x+y+z=N$ with positive integral coordinates. Triangles parameterize configuration space of triples of flags in $SL(N)$, plane algebraic curves of degree N, tropical plane curves. Triangles with numbers satisfying certain integrality conditions as well as some inequalities (such triangles are called hives) parameterize intertwiners of triple tensor products of irreducible representation of $SL(N)$. We shall show some connections between all these objects and also their relations to Young diagrams, combinatorial identities and Poisson geometry of Poisson-Lie groups.

K. Fukaya, *Mirror symmetry of toric A model and LG B-model.*

This is a joint work with Oh-Ohta-Ono

In this talk, I would like to explain the relationship between big quantum cohomology of (arbitrary) compact toric manifold with the universal unfolding of super potential. The later is shown to coincides with potential function with bulk deformation for Lagrangian fibers, which was introduced by us to study Lagrangian Floer theory. I will outline the proof that potential function with bulk deformation provides a universal unfolding of the super potential which is parameterized by a cohomology group of toric manifolds. I will show that K. Saito's structure of this universal unfolding coincides with Frobenius manifold structure of the big quantum cohomology of toric manifolds at least as a family of rings.

D. Gayet, *Deformations of associative submanifolds with boundary.*

Associative 3-submanifolds in a 7-dimensional manifold with a vector product can be considered as the generalization of complex curves. By Mc Lean, when their boundary is empty, the moduli space of their deformations is of vanishing virtual dimension. I will explain that when the submanifold is allowed to have a boundary in a fixed coassociative 4-submanifold, which can be viewed as the generalization of a Lagrange one, then the moduli space is of finite dimension, and in general non vanishing. Moreover, I'll give a sufficient geometric condition that forces the moduli space to be locally smooth.

M. Gross, *Tropical Geometry and Mirror Symmetry for* $\mathbb{P}^2$.

I will explain why tropical geometry for curve counting in $\mathbb{P}^2$ is equivalent in a strong sense to mirror symmetry for $\mathbb{P}^2$. In particular, since mirror symmetry for $\mathbb{P}^2$ also gives gravitational descendant Gromov-Witten invariants, we obtain tropical formulae for certain descendent invariants.

E. Katz, *Tropical Geometry and Monodromy.*

Tropical Geometry is a way of associating piecewise-linear objects called tropical varieties to algebraic varieties. These tropical varieties encode certain geometric properties of the algebraic varieties. I will explain how tropical geometry captures data about the cohomology of the algebraic variety, in particular about the monodromy action on the cohomology for a particular family of varieties. I will focus on the case of curves in detail. This is joint work with David Helm, Hannah Markwig, and Thomas Markwig.

F. Mangolte, *Automorphisms of real rational surfaces.*

Let X be a nonsingular real algebraic surface birational to $\mathbb{P}^2$. Denote by $\mathrm{Aut}(X)$ the group of real algebraic automorphisms of X. We prove that $\mathrm{Aut}(X)$ is dense in the group $\mathrm{Homeo}(X)$ of homeomorphisms of X into itself. The main ingredients of the proof are:

Th 1. $\mathrm{Aut}(S^2)$ is dense in $\mathrm{Homeo}(S^2)$, where S^2 is the unit sphere in $\mathbb{R}^3$.

Th 2. The mapping class group of a nonorientable rational surface is generated by the classes of real algebraic automorphisms.

This is joint work with J. Kollár.

H. Markwig, *Tropical Hurwitz numbers* (joint work with Renzo Cavalieri and Paul Johnson).

Hurwitz numbers count genus g degree d covers of $\mathbb{P}^1$ with fixed branch profiles. In this talk, we'll define a natural tropical analogue of Hurwitz numbers and show via cut and join equations that the tropical numbers agree with the classical numbers. This equality allows a new approach for open questions concerning Hurwitz numbers.

G. Mikhalkin, *Which tropical varieties are classically realizable?*

In tropical geometry the basic geometric objects (manifolds, varieties, cycles, etc) are defined intrinsically. In many but not all cases they can be approximated by 1-parametric families of classical (complex or real) objects so that the tropical object comes as the limit of this family. We review known results on local and global realizability.

E. Mukhin, *Applications of Bethe Ansatz to algebraic geometry.*

The gl_N Bethe algebra is a remarkable commutative subalgebra of the current algebra $Ugl_N[t]$. We argue that in many cases the representations of the Bethe algebra can be identified with the regular representations of the rings of the regular functions on suitable affine varieties. As an application, we obtain an effective proof that the Schubert Calculus holds over the field of real numbers by showing the transversality of intersections of Schubert cycles corresponding to some explicit real flags.

M. Shapiro, *Cluster algebras of finite mutation type and triangulations of surfaces.*

We establish basic properties of cluster algebras associated with oriented bordered surfaces with marked points. In particular, we show that the underlying cluster complex of such a cluster algebra does not depend on the choice of coefficients, describe this complex explicitly in terms of "tagged triangulations" of the surface, and determine its homotopy type and its growth rate.

Joint work with S. Fomin and D. Thurston.

Y. Soibelman, *Non-archimedean analytic geometry and generalized Donaldson-Thomas invariants.*

I am going to discuss the recent joint work with Maxim Kontsevich where we found an "abstract" definition of Donaldson-Thomas type invariants for 3d Calabi-Yau categories. It turns out that the generating functions for our invariants can be encoded in a non-archimedean analytic space. Moreover, the space of stability conditions of the Calabi-Yau category plays a role of the tropical degeneration (skeleton) of the analytic space.

J. Solomon, *An integrable hierarchy from open moduli.*

I will describe the structure of psi classes on open moduli space. I'll begin with background on Witten's conjectures governing closed psi classes. In the open theory, we find analogs of practically every phenomenon present in the closed theory. I'll exhibit open analogs of the string, dilaton, WDVV equations, the topological recursion relations, and the KdV integrable hierarchy. A common feature of much of this structure is that it intertwines the open and closed theories. I'll outline the definition of psi classes on open moduli space in genus 0.

D. Speyer, *Tropical Linear Spaces.*

We define tropical analogues of the notions of linear space and Plucker coordinate and study their combinatorics. The analogy to the classical case is very good: tropical Plučker vectors encode tropical linear spaces and intersections and orthogonal complements of tropical linear spaces behave as they ought. There is also an elegant combinatorial theory which relates to tropical linear spaces in the same manner that matroids relate to linear spaces, but it is difficult to determine which power series objects can be lifted to power series rings.

If time permits, we will also discuss the f-vector conjecture, which I have proven in many cases: All tropical linear spaces constructed from hyperplanes by repeated dualization and transverse intersection have the same f-vector and we conjecture that this f-vector is maximal for all tropical linear spaces of given dimension and codimension.

A. Varchenko, *The* (gl_n, gl_k)*-duality and critical points of master functions.*

The Bethe eigenvectors of the Gaudin model associated with a tensor product of representations of a Lie algebra are constructed using critical points of the associated master function.

The (gl_n, gl_k)-duality in representation theory provides a situation in which the Gaudin model associated with a tensor product of k representations of gl_n is isomorphic to the Gaudin model associated with a tensor product of n representations of gl_k. In this situation one expects a correspondence between the critical points of the two associated master functions.

I will explain this correspondence.

I. Zharkov, *Poincaré's formula in tropical jacobians.*

The self-intersections of theta divisor in Jacobians is homologically equivalent to the Abel-Jacobi image of (an appropriate multiple of) complementary powers of curves. It is known however that this is not true on the level of algebraic equivalence. I will discuss the analogous tropical problem.