ABSTRACT

Equivariant cohomology via relative homological algebra

Johannes Huebschmann
Université des Sciences et Technologies de Lille
Johannes.Huebschmann@math.univ-lille1.fr

We will explain how the appropriate categorical framework involving (co)monads and standard constructions provides categorical definitions of various relative derived functors including equivariant de Rham cohomology, Lie-Rinehart cohomology, Poisson cohomology, etc. This leads, in particular, to a description of equivariant de Rham theory as a suitable differential Ext over a category of modules relative to the group and the cone on its Lie algebra. Extending a decomposition lemma of Bott’s, we obtain a decomposition of the functor defining equivariant de Rham cohomology into two constituents, one constituent being the functor defining the appropriate relative Lie algebra cohomology and the other one that defining differentiable cohomology. For the case of a compact group, standard comparison arguments then lead to the familiar Weil and Cartan models and in particular explain why these models calculate the equivariant cohomology initially defined via a Borel construction. Pushing further the approach we arrive at a construction defining equivariant Lie algebroid or, somewhat more generally, equivariant Lie-Rinehart cohomology. This kind of construction provides, perhaps, a framework to explore constrained hamiltonian systems, the variational bicomplex, the Noether theorems and related topics. Interesting issues arise, e.g. how to define the cone in the category of Lie-Rinehart algebras or Lie algebroids. The question whether and how these constructions extend to Lie groupoids is momentarily open as is the question of existence of injective modules over Lie groupoids.

The talk will illustrate the formal approach, with an emphasis on the development of these ideas (Cartan, Weil, Cartan-Chevalley-Eilenberg, Cartan-Eilenberg, Mac Lane, Hochschild and collaborators, Bott and collaborators, ... ).