ABSTRACTS

WORKSHOP ON MATHEMATICAL TOOLS
April 15 – 17, 2009

Wednesday:

Suguru Arimoto
Modelling and control of multi-body mechanical systems: Part II Stability and control on Riemannian Constraint Submanifolds

Modeling, control, and stabilization of dynamics of two-dimensional object grasping by using a pair of multi-joint robot fingers are investigated under rolling contact constraints and arbitrariness of the geometry of the object and finger-ends. First, a fundamental testbed problem of modeling and control of rolling motion between 2-D rigid bodies with arbitrary shape is treated under the assumption that the two contour curves coincide at the contact point and share the same tangent. The rolling constraint induces the Euler equation of motion that is parameterized by a common arclength parameter and constrained onto the kernel space orthogonally complemented to the image space spanned from the constraint gradient. By extending the analysis to the problem of stable grasp of a 2-D object with arbitrary shape by a pair of robot fingers, the Euler-Lagrange equation of motion of the overall fingers/object system parameterized by arclength parameters is derived, together with a couple of first-order differential equations that express evolutions of contact points in terms of the second fundamental form. It is shown that 2-D rolling constraints are integrable in the sense of Frobenius even if their Pfaffian forms are characterized by arclength parameters. A control signal called ``blind grasping'' is introduced and shown to be effective in stabilization of grasping without using the details of object shape and parameters or external sensing. An extension of Dirichlet-Lagrange's stability theorem to a class of systems of DOF-redundancy under constraints is suggested by using a Morse-Bott function. Finally, suggested by the arguments dedicated to modeling of 2-D grasping, mathematical tools for modeling of 3-D grasping of a rigid object with arbitrary shape under rolling constraints are introduced and an implicit form of Euler-Lagrange's equation of motion of the overall system is derived.

Andreas Rauh
Interval Methods for Verification and Implementation of Robust Controllers

In recent years, powerful interval arithmetic tools have been developed which allow for computation of guaranteed enclosures for the sets of reachable states of dynamical systems described by ordinary differential equations (ODEs) as well as differential algebraic equations (DAEs). In these simulations, uncertainties in initial conditions and parameters are taken into account by worst-case bounds. The resulting state enclosures are guaranteed in the sense that all reachable states are certainly included. The enclosures take into account both the influence of the abovementioned uncertainties as well as numerical inaccuracies arising from computer implementations using floating-point arithmetic [3].
In addition, verified routines exist for the approximation for optimal control strategies for discretetime as well as continuous-time systems with uncertainties. Besides the consideration of robustness requirements, these routines provide verified information about the worst-case distance of an approximate solution from the global optimum. The computational framework, that has been developed, makes use of the C++ library PROFIL/BIAS for an implementation of basic interval arithmetic functionalities and VALENCIA-IVP for verified solution of initial value problems for both ODEs and DAEs [1].
Since techniques for algorithmic differentiation are readily available in software packages such as FADBAD++, it is possible to verify the prerequisites for applicability of specific control design approaches such as exact input-output linearization or flatness-based control. The basic methods that are

used in this case are the verified evaluation of sufficient criteria for reachability and observability of states on the one hand and asymptotic stability on the other hand. Routines for verified stability analysis directly yield information about guaranteed bounds for the region of attraction of an asymptotically stable equilibrium [2].
Additionally, the application of DAE solvers to state, parameter, and disturbance estimation in open-loop and closed-loop control systems is discussed. For both ODEs and DAEs, this task can be formulated with the help of DAEs if measured data are available. Finally, trajectory planning and computation of control strategies using verified DAE solvers is addressed. In this case, input variables of a dynamical system are determined using verified computational procedures in such a way that the output signal matches a predefined time response within given tolerances. For that purpose, relations between criteria for solvability of DAEs and criteria for controllability, reachability, and observability of states that are well-known in control theory are investigated. Selected applications are presented to visualize the interval arithmetic techniques. Examples for implementation of interval methods in rapid control prototyping environments conclude this talk to demonstrate possibilities for real-time implementations.

Dong Eui Chang

The Energy Shaping Method for Stabilization of Mechanical Systems

The energy shaping method is effective in designing controllers to stabilize mechanical systems. The basic idea in this method is simple: given a mechanical system, we find a feedback-equivalent mechanical system whose energy obtains a minimum at an equilibrium point of interest so that the shaped energy can be used as Lyapunov function. In a broad sense, this is a form of the passivity theory restricted to mechanical systems. In this talk, we discuss some criteria for Lyapunov/asymptotic/exponential stabilization by energy shaping, which can be verified without solving any PDE's that are normally involved in the energy shaping method. We then present how to shape an external force into a form of dissipative force in order to let the shaped energy decrease in time. Finally, we illustrate the method with some examples.

Laurent Praly

Observers: Some general remarks and more advanced topics

State estimation has received attention by researchers for more than 60 years and is still a very active area. But often when going for such a long way, the very first problem statement may have been forgotten. To make sure this is not the case, we start our presentation by some well known facts like: without uncertainty the observation problem is "trivial" as soon as we have some kind of observability. We continue with some more recent contributions like high gain observers with gain adaptation or the extension of the (exact) Luenberger observer to the
nonlinear case and present some applications.

Markku Nihtilä

WeiNorman technique for control design of bilinear ODE systems with application to quantum control

A two-level quantum system model describing population transfer driven by a laser field is studied. A four-dimensional real-variable differential equation model is first obtained from the complex-valued two-level model describing the wave function of the system. Due to bilinearity in the control and the states Lie-algebraic techniques can be applied for constructing the state transition matrix of the system. The Wei-Norman technique is used in the construction. The exponential representation of the transition matrix includes three base functions, two of which serves as the parameter functions, which can be chosen freely. This corresponds to considering the overall control system as an underdetermined differential system. In this framework the initial and final states can be defined corresponding to the two levels of the original system model. Then flatness-based design is applied for explicitly calculating the parameter functions, which in turn give the desired input-output pairs. This input then drives the state of the system from the given initial state to the given final state in a finite time.

Sonia Schirmer
Hamiltonian and Markovian Reservoir Engineering for Quantum Systems

Hamiltonian engineering has been shown to be a powerful technique, which can be applied to many different problems that involve steering a quantum system to achieve a desirable outcome, and a particularly promising approach to Hamiltonian engineering is the optimal control approach, i.e., formulating the problem as an optimization problem. However, the problem formulation is important, and although optimization is a well-established field, the solution of the resulting optimization problems is usually not trivial, in part because the search space is usually infinite dimensional. To overcome this obstacle the controls must be parameterized, and the parameterization is critical. The most common approach is to approximate the controls using piecewise constant functions. While adequate for some problems, such a parameterization inevitably leads to high bandwidth solutions due to the discontinuities of the fields. We demonstrate that using more natural parameterizations we can significantly reduce the bandwidth of the fields, although at the expense of having to solve more complex optimization problems. Another crucial variable is the problem formulation itself. Often, optimal control problems are formulated using Hamiltonians that incorporate many approximations, e.g., RWA, off-resonant excitations and fixed couplings negligible, etc, which inevitably limit what can be achieved by optimal control. We show that we can in principle speed up the implementation of quantum gates several orders of magnitude compared to conventional frequency-selective geometric control pulses for certain systems by avoiding such approximations and taking advantage of the full range of off-resonant excitations and couplings available in the optimal control framework. Another problem with Hamiltonian engineering is that the most effective approaches are model-based, i.e., we require a model of the system, especially its response to external fields, or the functional dependence on the controls. In some cases this isn't a problem and optimal controls can be designed to be robust with regard to model uncertainties. For other problems, however, such as information transfer through spin networks using simple local actuators, it can be shown that the optimal switching sequences are highly model-dependent, while the exact network topology and precise couplings for such systems are usually not known. Such problems call for closed loop optimization. We show that we can effectively solve problems such as finding optimal switching time sequences for such networks by adapting gradient-based optimization algorithms even for problems where the standard evolutionary algorithms fail completely to find acceptable solutions. Finally, there are certain types of problems that Hamiltonian engineering, although an extremely powerful tool for quantum engineering, cannot solve. One such problem is stabilization in the presence of environmental interactions. This problem can in principle be addressed using reservoir engineering.
We consider a variant of Markovian reservoir engineering using direct feedback from an indirect measurement such as homodyne detection. We show that if the control and feedback Hamiltonians in this setting are unrestricted and we have some degree of control over the type of measurement we can perform, then any state can be in principle be stabilized.

Mazyar Mirrahimi
Real-time feedback control in quantum optics

In this presentation, after a brief introduction on different types of models in quantum physics and their relevance with respect to the considered tasks, we will concentrate on two experimental settings in quantum optics. The main problem for the two experiments concerns the robust preparation and protection of a fully quantum state. To this end, we will apply quantum feedback schemes to the Markovian random processes driving the measured systems. The feedback law is designed applying stochastic Lyapunov techniques and its performance and robustness is tested through Monte-Carlo simulations. The proof of the convergence results will be sketched applying the Kushner's invariance theorem and some other probabilistic tools. A more detailed analysis of the two problems can be find within the references [2, 1]

Thursday:

Andrei Agrachev
Smooth optimal synthesis for infinite horizon variational problems.

We give an effective sufficient condition for a variational problem with infinite horizon on a Riemannian manifold M to admit a smooth optimal synthesis, i. e. a smooth dynamical system on M whose positive semi-trajectories are solutions to our variational problem. To realize the synthesis we construct a well-projected to M invariant Lagrange submanifold of the extremals` flow in the cotangent bundle T*M. The construction uses the curvature of the flow in the cotangent bundle and some ideas of hyperbolic dynamics.

Peter J. Vassiliou
Contact and Control

In this talk I will discuss aspects of the geometry of jet spaces that are relevant to control theory. In particular, I will present a new geometric characterisation for partial prolongations of the jet space of maps from the real line. Time permitting; I will describe the application of this result to control systems for curves in homogeneous spaces and discuss links to Cartan prolongation and differential flatness.

José De Dona
Constraints in control problems

In this talk we will discuss some topics concerning constrained control in relation with differential flatness. It is well known that the differential flatness framework allows for a complete parameterisation of a system trajectories by a particular set of variables, known as flat outputs, in such a way that the system dynamics become trivial. However, the transformations of variables that allow for such a trivialisation of the trajectories are, in general, highly nonlinear. As a consequence, the presence of inequality constraints on system variables (inputs and/or states), implies that constraints that are relatively innocuous in the original problem, can give rise to highly non trivial problems in the transformed space. Typical solutions that have been attempted in the past include approximations of the constraints and parameterisation of the flat outputs by restricting attention to particular basis functions, e.g., polynomials, splines, etc. Some of these techniques tend to be conservative since they do not capture all the possible constrained trajectories of a system. In the line of research discussed in this talk we aim at parameterizing all possible trajectories of a constrained control problem. In this context, and trying to understand the nature of constrained problems, we will discuss the concept of admissible sets of states. Such sets turn out to be closed under quite general assumptions, and hence the study of their boundary is of particular interest, since these objects would, in most cases, completely characterise these sets. The talk will concentrate on some of the properties of these boundaries and possible ways to characterise them. Since the topic is relatively new to the authors, it is not claimed that all possibly related existing results will be conveyed. However, some connections with optimal control, reachability and viability theory will be discussed.

Peter J. Olver
Moving Frames in Applications

In this talk, I will describe a new approach to the powerful method of moving frames that applies to very general Lie group actions. I will discuss several new directions, concentrating on the analysis of invariant variational problems and invariant submanifold flows, with applications to mechanics, computer vision and integrable systems.

Jon M. Selig
Rational Interpolation of Rigid-body Motions

The group manifold of the group of rigid-body displacements can be considered as an open set in a six-dimensional non-singular quadric, known as the Study quadric. Rational motions are rational curves in this quadric. Using a birational map the quadric can be transformed to six-dimensional projective space $\Pe^6$. These birational maps are simply derived from Cayley maps from the Lie algebra of the group to the group itself. In this way interpolation problems in the group can be transformed into interpolation problems in the Lie algebra. These interpolation problems are important in Robotics and Computer Graphics applications. Three simple examples will be considered; simple linear interpolation, Hermite interpolation with velocity constraints at the endpoints and interpolation using Bennett biarcs.

Zhong-Ping Jiang
Nonlinear control of underactuated systems: some new problems

In this talk, some challenging control problems are addressed for underactuated mechanical systems which enjoy more degrees of freedom than the number of controls. Practical examples of underactuated systems are abundant and, to name only a few, include satellites, mobile robots, surface ships and underwater vehicles. Finding solutions to control problems for these systems is an important task. But more importantly, we argue that this class of nonlinear dynamical systems provides us with an opportunity to search for new tools for modern nonlinear control.

To approach this end, we will look at a new problem, which we name as "simultaneous stabilization and tracking", for mobile robots and surface ships. Roughly speaking, we aim at finding a single (smooth) controller that can solve both stabilization and tracking at the same time. A conventional viewpoint in most control textbooks is that stabilization is a special case of tracking. However, as several authors have remarked, this is not quite so for underactuated mechanical systems. Indeed, it is exactly for this reason that stabilization and trajectory-tracking have been studied separately by many authors over the last 20 years. We will report on our preliminary results on this issue of simultaneous stabilization and tracking and other problems such as output feedback control. We expect that our research can entice younger graduates to this area and to search for new mathematical tools for general nonlinear control systems.

Friday:

Elizabeth Mansfield
Introduction to the invariant calculus with application to Variational Problems with Symmetry

The reformulation of the concept of moving frames by Fels and Olver allows it to apply to many classes of equivalence problems that do not necessarily arise in differential geometry. Suppose that a Lie group G acts smoothly on some smooth space M. A moving frame is defined to be an equivariant map from M to G. The equivariance, together with undergraduate multivariable calculus, allows for a plethora of results concerning the symbolic manipulation of invariants to be obtained. Applications that interest the speaker include solution methods for differential equations, both analytic and numerical, and the analysis of Euler Lagrange systems for both smooth and discrete variational problems. In this talk, I will outline the `landscape' of the symbolic invariant calculus and show how one may obtain insight into the structure of Euler Lagrange systems arising from a smooth variational problem with a Lie symmetry group. We will also give a new result concerning the structure of the first integrals guaranteed by Noether's Theorem. The central differential-algebraic concept used is that of differential relations or syzygies between differential invariants. The whole point is that these may be computed symbolically knowing only the infinitesimal action and the equations defining the frame; that is, without knowing either the invariants or the frame explicitly.

Alban Quadrat
New advances in the computation of flat outputs of flat linear functional systems

Using on a constructive version of the Quillen-Suslin theorem and its implementation in the package QUILLEN_SUSLIN [2], we first present recent advances in the computation of at outputs of at linear ordinary di_erential time-delay systems. In particular, we prove that a linear ordinary differential time-delay system is equivalent to the controllable ordinary differential system obtained by setting all the delay amplitudes to 0 [2]. Hence, a linear ordinary differential time-delay system is Lie-Bäcklund equivalent to a Brunovsky linear system as is the case for at nonlinear ordinary differential systems. Moreover, we prove that every controllable linear ordinary differential system with polynomial coefficients and at least two inputs is globally at, i.e., admits a global injective parameterization without singularities. Based on a constructive version of Stafford's theorem and its implementation in the package STAFFORD, we shall present a constructive algorithm which computes injective parameterizations and flat outputs [3]. We shall discuss the extension of those results to linear analytic systems and systems of partial differential equations.
Finally, we shall explain how the previous results can be used to reduce and decompose linear functional systems (e.g., ordinary differential time-delay systems, systems of partial differential equations) [1]. Using the package ORE_MORPHISM, we shall show that many examples studied in the literature of control theory can be reduced or decomposed [1], which highly simplifes the study of their structural properties (e.g., controllability, observability, flatness, pi-freeness)

Kurt Schlacher
Construction of flat outputs by reduction and elimination

The construction of flat outputs for nonlinear lumped parameter systems is still a challenging problem in the general case, although significant progresses have been made in the last years. An algorithm, based on successive reduction of the number of variables and elimination of variables, equivalent to a reduction of the number of equations, is presented. Since the structure of the reduced system is essential, whether further simplifications are possible, the geometric structure of undetermined systems of implicit differential equations will be studied. It will turn out, that the dynamic extension of the system, such that it is transformable to a form affine in the derivative coordinates, is the essential tool. Locally necessary and sufficient condition for the existence of such an extension, which leads to a reduction, will be presented. The geometric interpretation is that certain vector-fields become projectable on the manifold defined by the extended system or that a certain distribution is stable with respect to a covariant derivative. In addition, the use of the derived flag of an explicit control system allows us to give a straightforward interpretation of the properties presented above. Finally some applications of this approach are presented.

Michel Fliess
Model-free-control and "intelligent" PID controllers: Towards a possible trivialization of nonlinear control?

We are introducing a model-free control and a control with a restricted model for finite-dimensional complex systems. This control design may be viewed as a contribution to ``intelligent'' PID controllers, the tuning of which becomes quite straightforward, even with highly nonlinear and/or time-varying systems. Our main tool is a newly developed numerical differentiation. Differential algebra provides the theoretical framework. Our approach is validated by several numerical experiments.

Felix Antritter

Flatness necessary and sufficient conditions and their evaluation

This talk deals with the characterization of differentially flat nonlinear control systems in implicit representation, using the differential geometric framework of jets of infinite order. Necessary and sufficient conditions for differential flatness are derived by studying the properties of the differentials of Lie-Bäcklund transformations which transform the system under consideration into a trivial system.
As a result, an algorithm for the computation of flat outputs can be derived. The algorithm consists of two steps: At first a flat output of the variational system is computed using a polynomial matrix approach. Then, a flat output of the nonlinear system can be constructed if the ideal generated by the flat output of the variational system is strongly closed. This can be checked using the generalized moving frame structure equations.
The evaluation of the conditions using standard computer algebra systems is discussed and all steps of the algorithm will be illustrated by means of an example. Additionally some recent results will be presente

Joachim Rudolph

Trajectory planning for the control of distributed parameter systems with lumped controls

The design of feed-forward and feedback control for finite dimensional systems and delay systems is largely simplified by flatness, respectively freeness. This has been shown in numerous academic case studies and industrial applications. A central part in the control design (the importance of which has often been under-estimated) is trajectory planning. It is particularly useful for distributed parameter systems with lumped control inputs, a class of systems the models of which include partial differential equations. In the linear case, as for delay systems, a module-theoretic framework has been established, and the trajectory planning is based on the use of a module basis, which plays a role similar to the one of a flat output in finite-dimensional flat systems. Examples of distributed parameter systems studied are heat conductors, elastic piezo-beams and plates, elastic robot arms, ropes, electric lines, tubular chemical reactors, and heat exchangers. Although many of the problems considered are linear with fixed boundary, some nonlinear and free boundary value problems have been solved, too. The present talk provides a tutorial introduction to the “flatness-based control” of distributed parameter systems with lumped controls. Emphasis will be placed on trajectory planning as part of the control design and physically meaningful specific examples.